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$$\begin{aligned}
& + 2\log[\theta + \sqrt{(2 + \theta^2)}] \Big]_{\theta=0}^{\theta=60} = 3\sec\alpha \left\{ \frac{1}{2} [60\pi\sqrt{(2 + 3600\pi^2)} \right. \\
& \quad \left. + 2\log[60\pi + \sqrt{(2 + 3600\pi^2)}] - \frac{1}{2} [0\sqrt{2} + 2\log(0 + \sqrt{2})] \right\} \\
& = 3\sec\alpha \{ 30\pi\sqrt{(2 + 3600\pi^2)} + \log[60\pi + \sqrt{(2 + 3600\pi^2)}] - \log\sqrt{2} \} = 4540 \text{ feet.}
\end{aligned}$$

### DIOPHANTINE ANALYSIS.

116. Proposed by HARRY S. VANDIVER, Bala, Pa.

If  $n$  is an odd positive integer, and  $1, n, n', n'', \dots$  denote all its distinct divisors, then  $2^n > 2[n+1][n'+1][n''+1]\dots$

Solution by L. E. DICKSON, Ph. D., Assistant Professor of Mathematics, The University of Chicago.

There is a single exception  $n=3$ , for which  $2^3=2[3+1]$ . The corrected theorem may be proved by induction, using the following lemma:

If  $p$  is an odd prime number and  $d$  and  $\pi$  positive integers,

$$[1+d]^{p^\pi} > [1+d][1+pd][1+p^2d]\dots[1+p^\pi d],$$

except for  $d=1, \pi=1, p=3$ , the equality sign then holding.

For proof we apply  $p^\pi - 1 = p - 1 + p[p-1] + p^2[p-1] + \dots + p^{\pi-1}[p-1]$ .

$$\therefore [1+d]^{p^\pi} = [1+d][1+d]^{p-1}[1+d]^{p(p-1)}\dots[1+d]^{p^{\pi-1}(p-1)}.$$

But  $[1+d]^{t(p-1)} \geq [1+td]^{p-1} \geq 1 + [p-1]td + [td]^{p-1} \geq 1 + [p-1]td + td$ , if  $p > 2$ . The equality signs hold simultaneously only when  $t=d=1, p=3$ . Hence, for  $v > 2$ ,  $[1+d]^{t(p-1)} > 1 + ptd$  unless  $t=d=1, p=3$ , so that the lemma follows.

To prove the theorem by induction, we note that it is true for  $n=p^\pi > 3$ , in view of the lemma for  $d=1$ . Assume that it has been verified for  $n=p_1^{\pi_1}p_2^{\pi_2}\dots$ . We proceed to prove it true for  $N=np^\pi$ ,  $p$  being prime to  $n$ . We have

$$2^N = [2^n]^{p^\pi} \geq [(1+1)(1+n)(1+n)\dots]^{p^\pi}$$

$$\geq [(1+1)(1+p)\dots(1+p^\pi)][(1+n)(1+pn)(1+p^2n)\dots][(1+n')(1+pn')\dots],$$

in view of the lemma. But the distinct divisors of  $N$  are

$$1, n, n', \dots, p, pn, pn', \dots, p^2, p^2n, \dots, p^\pi, p^\pi n, \dots$$

The theorem is therefore true for  $N$ .

118. Proposed by L. C. WALKER, A.M., Professor of Mathematics, Colorado School of Mines, Golden, Col.

Find the two least integral numbers such that their sum shall be a square and the sum of their squares a biquadrate.

Solution by G. B. M. ZERE, A. M., Ph. D., Parsons, W. Va.

Let  $x$  and  $y$  be the numbers, then for  $x+y=1, x^2+y^2=13^4$ ,  
 $x=120, y=-119$ .